SRSVIEW
Shock Response Spectrum Analysis

Introduction

Shock Response Spectrum (SRS) analysis was under development as a standard data processing method in the early 1960's. SRS was initially used by U.S. Department Of Defense engineering contractors and government R&D facilities. Dr. Irwin Vigness of the U.S. Navy Research Lab did much to establish the method as an indicator of mechanical shock severity.

There is not quite the same motivation to apply the method today, because modern computer and signal processing technology makes it possible to directly compute the response of specific structures resulting from well defined actual transient forces. Nevertheless, the SRS analysis method continues to grow in popularity with vibration test labs and analytical organizations. The method is even gaining in popularity for steady state vibration applications as well as transients. NASA routinely applies the method for evaluating the severity of Orbiter Payload Bay vibration to new payloads.

Example Drop Shock Event

The SRS concept will be developed in the following context. Consider a product that is produced in quantity consisting of a dynamic base structure on which many smaller dynamic components are mounted. The structure could be any physical size, just so the base structure is large compared to the components to be mounted on it. The base structure could be an electronics chassis structure or a NASA Space Shuttle. From time to time new components will be designed for installation in the system. Also, the base structure is occasionally modified. This leads to a need for the separate testing and analysis of the base structures and the components. Consequently, we encounter two different points of view in the application of the SRS method, one that focuses on the design and testing of base structures and one that focuses on the design and testing of components to be mounted on base structures

Now, suppose that, as a consequence of either its environment or normal operation, a vibration transient is generated on the base structure. Many sources of transients could be cited: Drop impact during handling, explosive bolts on aerospace structures, reciprocating engine fuel explosions inside cylinders, bolted joints suddenly opening and closing with an impact, etc. Let's develop and example for which measurement of the acceleration vs. time transient has been performed. And for the sake of the example, assume that the transient actually results from the product dropping onto a concrete surface, producing a half-sine impact load on one corner of the base structure. This provides the basis for applying the SRS test and analysis method.

Imagine that during the original design of the product a Finite Element Model (FEM) of the base structure was developed. An idealized simple representation of the components was included in the model. The large numbers of components and their explicit complexity preclude a detailed model of the components at this stage. A critical design requirement is that the product survives the drop impact event. The design engineer iterates on the design of the base structure until the best compromise is reached among the various competing design requirements imposed on him. The design features externally cushioned corners that provide for some degree of shock isolation during drop events. The designer finds that applying a half-sine acceleration-time function to his final FEM design approximately simulates a worst case drop impact event. This half-sine transient has a peak acceleration level of 175 G and duration of 10 milliseconds. Figure 1 represents the analytical acceleration transient (G versus time) applied to the FEM.

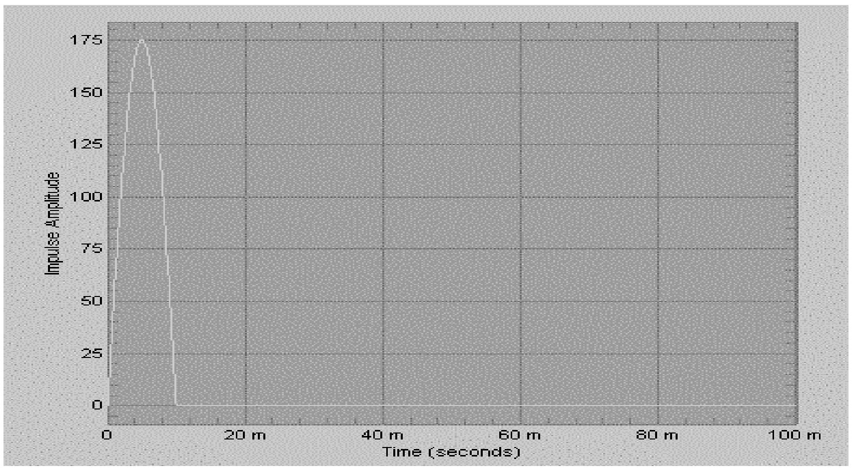

Figure 1. A Half-Sine acceleration transient applied to a base frame structure simulating a drop shock event.

When the analytical half-sine pulse is applied to the FEM, a response acceleration transient is computed for points around the structure where components are mounted. The measurements are not made directly on the components, but adjacent to component attachment points. A location is chosen to represent the shock environment that components must be designed to withstand. The transient response acceleration at that point is used to define the shock environment for components. Furthermore, once this response acceleration has been established for the final design, any future modifications of the base frame structure must not generate a shock environment any more severe than this. Thus, we have an acceleration response transient that will be used in some way to establish a specification for the design and test of base structures and also any components to be mounted on the base structure.

The computed response acceleration is shown in Figure 2. The base frame has been designed free of major vibration response to the degree practical, but it is apparent that some base frame resonances have been excited.

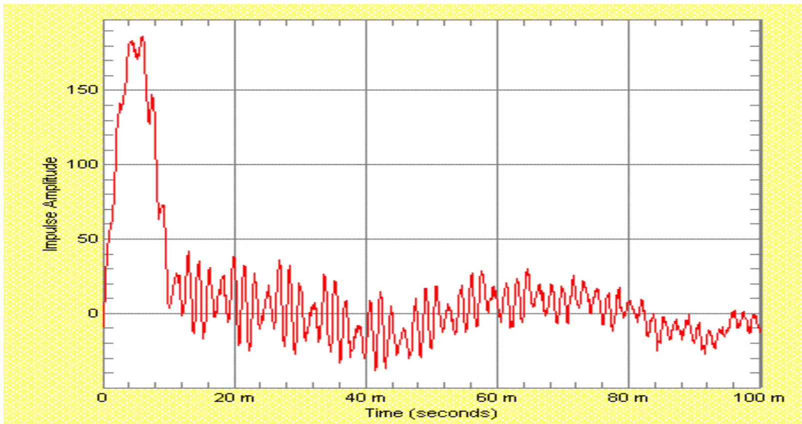

Figure 2. Response acceleration transient for the base frame example structure. This represents the shock environment for the components mounted on the base frame.

SRS As A Base Structure Specification

Let's first consider the role of the SRS (Shock Response Spectrum) as a specification for the design and testing of the example base frame structures. There are two separate kinds of requirements to consider in designing for the drop shock event. It is obvious that the base frame must be designed to avoid structural failure under the applied shock load. But the requirement we wish to focus on here is that of designing the base frame structure so that it will not respond to the drop shock with acceleration levels that will damage the components.

We seek a specification that will allow evaluation of production base frame structures for component-safe response vibration. The base frame structure will be tested by applying the half-sine acceleration transient to one corner. We would like the specification to be based on the measured acceleration response at the standard component attachment position. We have the original base frame transient as a reference, representing the acceptable design. The problem is that it is not necessarily relevant to just compare the transient produced on a new test unit to the reference transient. Without knowledge of how components will be affected by the new transient, there is no basis for establishing a tolerance band for the transient.

What about the PSD (Power Spectral Density) or maybe the Fourier Transform of the transient? You still have the same basic problem. You don't know what effect a change in spectrum amplitude at a given frequency will have on the components. Additionally there is the question, what time period would you select for the Fourier Transform? Over what time duration would you average the PSD? The SRS bypasses all of these problems and provides a natural indicator of relative shock severity.

One of the things that keeps us from knowing how severe a shock transient will be for components mounted on the base structure is that we generally don't know what the dynamic characteristics of all of the components are. The SRS method basically says that it doesn't matter. It doesn't matter what resonance frequencies are represented by the components, because we will simply apply the shock transient from the base frame to every conceivable SDOF (Single-Degree-Of-Freedom) structure that could be mounted on the base frame. Thus, in the absence of actual models for all of the components, a standard set of components is made available.

Imagine a platform with 1000 different SDOF components mounted on it. Each component has a different resonance frequency, so that every resonance frequency of possible interest is represented. We have the equivalent of such a system when applying the SRS method. We enforce on that platform the acceleration transient that was present on our base frame. Each of the SDOF components will respond with its own unique acceleration transient. The peak response acceleration level is then computed for each SDOF component. The set of all peak levels is seen to be representative of the severity of the base frame shock transient. This set of peak levels can be collected together to form a spectrum across the frequency range of interest. This is the SRS. The process is pictured in Figure 3.

There are a number of ways to compute the SRS, including use of recursive digital filters, FIR filters, Frequency Response Functions and directly solving SDOF differential equations using the Runge Kutta method. Signalysis follows a procedure based on the paper by David Smallwood of Sandia National Laboratories, Albuquerque, New Mexico, "An Improved Recursive Formula For Calculating Shock Response Spectra." This paper was published in The Shock And Vibration Bulletin, Bulletin No. 51, Part 2, May 1981.

All of the methods generate a time series solution equivalent to solving the second order differential equation,

$$m\ddot{y}(t) + c\dot{y}(t) + ky(t) = \ddot{z}(t) \tag{1}$$

The differential equation of equation (1) represents the situation sketched in Figure 4 and Figure 5. The input base displacement is z(t) and the response displacement of the mass is u(t). A single dot over a variable indicates velocity (first derivative) and double dots over a variable indicate acceleration (second derivative). The response motion to be solved in equation (1) is the difference displacement, y(t), between the mass response motion, u(t), and the input displacement, z(t):

$$y(t) = u(t) - z(t) \tag{2}$$

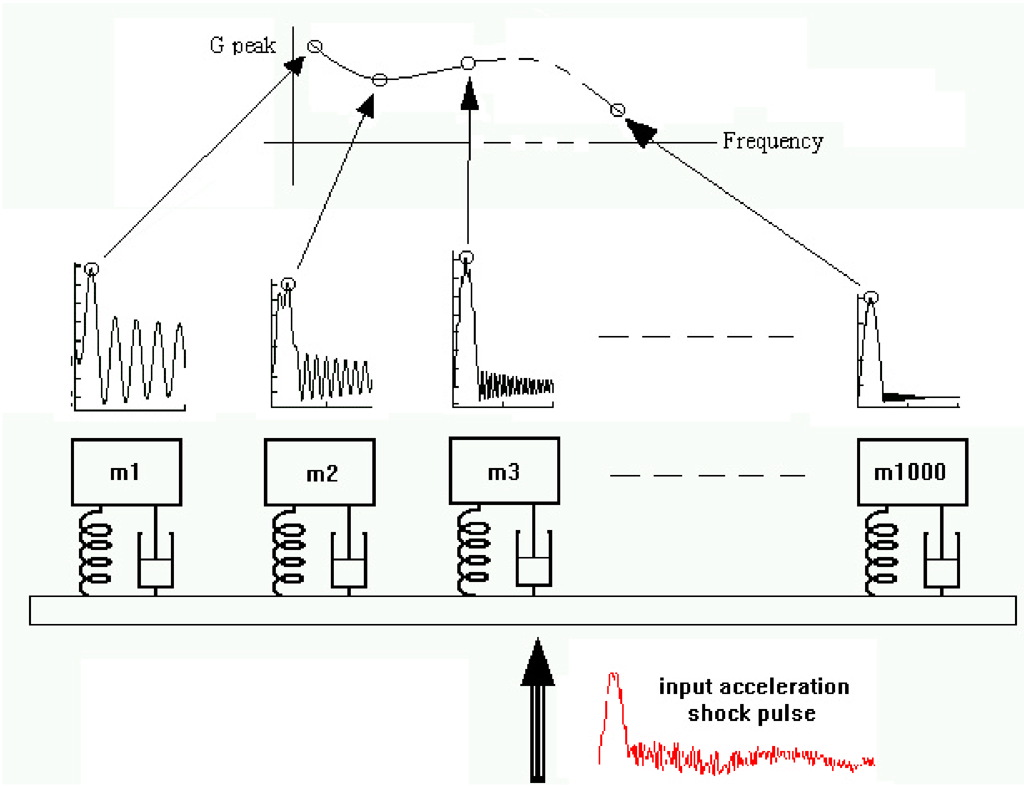

Figure 3. The SRS (Shock Response Spectrum) concept. An input acceleration transient to be analyzed is processed mathematically in a way that simulates the process represented here.

Likewise, the response velocity of equation (1) is

$$\dot{y}(t) = \dot{u}(t) - \ddot{z}(t) \tag{3}$$

and the response acceleration is

$$\ddot{y}(t) = \ddot{u}(t) - \ddot{z}(t) \tag{4}$$

The differential equation (1) results from the following considerations. The free body diagram of Figure 5 identifies the external forces applied to the mass: The spring force, F_k and the damping force, F_c. Setting the sum of forces acting on the mass equal to mass times acceleration (Newton's second law):

$$F_k + F_c = m\ddot{u} \tag{5}$$

The spring is linear and obeys Hook's Law. Notice that positive (upward) motion of the base compresses the spring into the mass above, pushing it upward with positive force. But any upward motion of the mass, i.e., displacement in u, tends to stretch the spring, resulting in spring tension pulling the mass in the downward, negative direction.

$$F_k = -k(u - z) \tag{6}$$

The damping force is proportional to velocity due to the viscous damper with damping constant, c:

$$F_c = -c(\dot{u} - \dot{z}) \tag{7}$$

Substituting (6) and (7) into (5), we have the second order differential equation in u(t):

$$-m\ddot{u} - c(\dot{u} - \dot{z}) - k(u - z) = 0 \tag{8}$$

Set up equation (8) for a change in variable by adding the mass times base acceleration to both sides:

$$-m(\ddot{u} - \ddot{z}) - c(\dot{u} - \dot{z}) - k(u - z) = m\ddot{z} \tag{9}$$

Making the change of variable, y = u-z, and the corresponding derivatives, as indicated in equations (1), (2) and (3), and interchanging signs on both sides of equation (9),

$$m\ddot{y}(t) + c\dot{y}(t) + ky(t) = -m\ddot{z} \tag{10}$$

This is the same as equation (1). Notice that the base acceleration term on the right side plays the role of a forcing function.

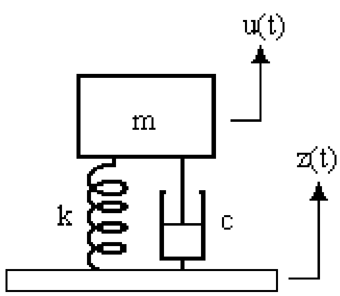

Figure 4. A SDOF system responding with motion u(t) to base motion z(t).

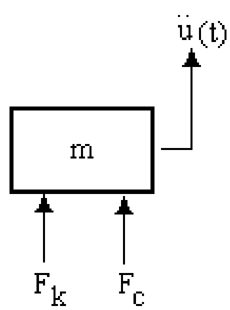

Figure 5. Free body diagram for the SDOF mass. Forces are applied through the spring and damper.

Fourier Transforming both sides of equation (10) gets the differential equation into the frequency domain.

$$m\ddot{Y}(w) + c\dot{Y}(w) + kY(w) = -m\ddot{Z}(w) \tag{11}$$

At each frequency there is a simple algebraic relationship between displacement, velocity and acceleration. Substituting these relationships allows all terms on the left to be expressed in terms of displacement-frequency functions. And factoring out the common displacement factor we now have a relatively simple algebraic equation in place of a differential equation.

$$[-w^2 m + iwc + k]Y(w) = -m\ddot{Z}(w) \tag{12}$$

Divide equation (12) by the mass, m.

$$[-w^2 + iw\frac{c}{m} + \frac{k}{m}]Y(w) = -\ddot{Z}(w) \tag{13}$$

The critical damping for a SDOF system is

$$c_c = 2\sqrt{km} \tag{14}$$

The damping ratio, ζ, is

$$z = \frac{c}{c_c} = \frac{c}{2\sqrt{km}} \tag{15}$$

The resonance frequency as radians per second is

$$w_r = \sqrt{\frac{k}{m}} \tag{16}$$

The ratio of frequency to resonance frequency, β, will be used, that is

$$b = \frac{w}{w_r} \tag{17}$$

Equations (15) and (16) may be used to yield the relation,

$$\frac{c}{2m} = w_r z \tag{18}$$

Making substitutions using the relations of equations (14) through (18) and rationalizing the denominator after dividing through by the factored expression on the left side of equation (13) leads to the solution for Y(ω).

$$Y(w) = \frac{-\ddot{Z}(w)[(1-b^2) - i2zb]}{w_r^{\,2}[(1-b^2)^2 + 4z^2 b^2]} \tag{19}$$

Vibration test lab engineers and dynamics analysts alike favor the use of the ratio of response acceleration to input acceleration. This ratio is known as Transmissibility, T(ω), also referred to as the amplification factor. Solving for Transmissibility in equation (19),

$$T(w) = \frac{\ddot{Y}(w)}{\ddot{Z}(w)} = \frac{b^2[(1-b^2) - i2zb]}{(1-b^2)^2 + 4z^2 b^2} \tag{20}$$

SRSVIEW allows the user to enter SDOF parameters that specify the damping, frequency range and number of SDOF systems to be included in the analysis. The damping is specified as a decimal number corresponding to the damping ratio, ζ.